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Sri Lanka Institute of Information Technology

IT0060 –Essential Mathematics

Ms. Pubudika Wijesundara
Ms. Shashini Kumarasinghe

Differentiation

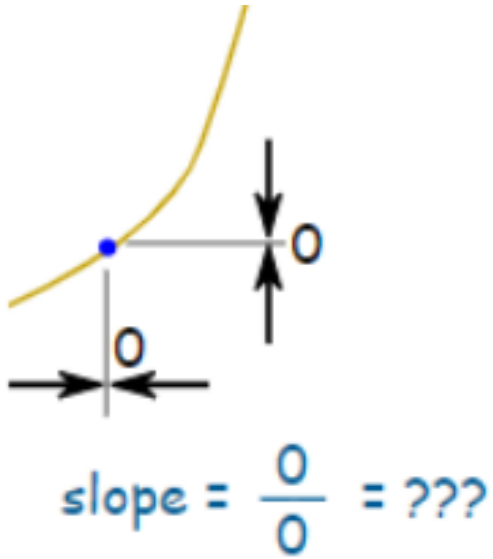
Outline

- Differentiation
  - Definition of derivative
- Basic differentiation rules
  - Power rule
  - Sum and constant rules
- Derivatives of polynomial functions

Differentiation

In Differential Calculus, we learn how to find the "gradient at a point".

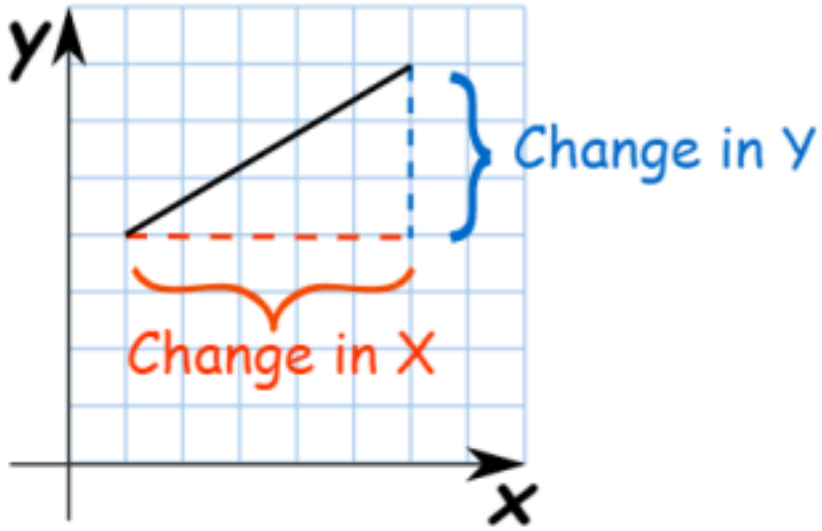
Gradient >> **Slope**



Differentiation: Gradient of a straight line

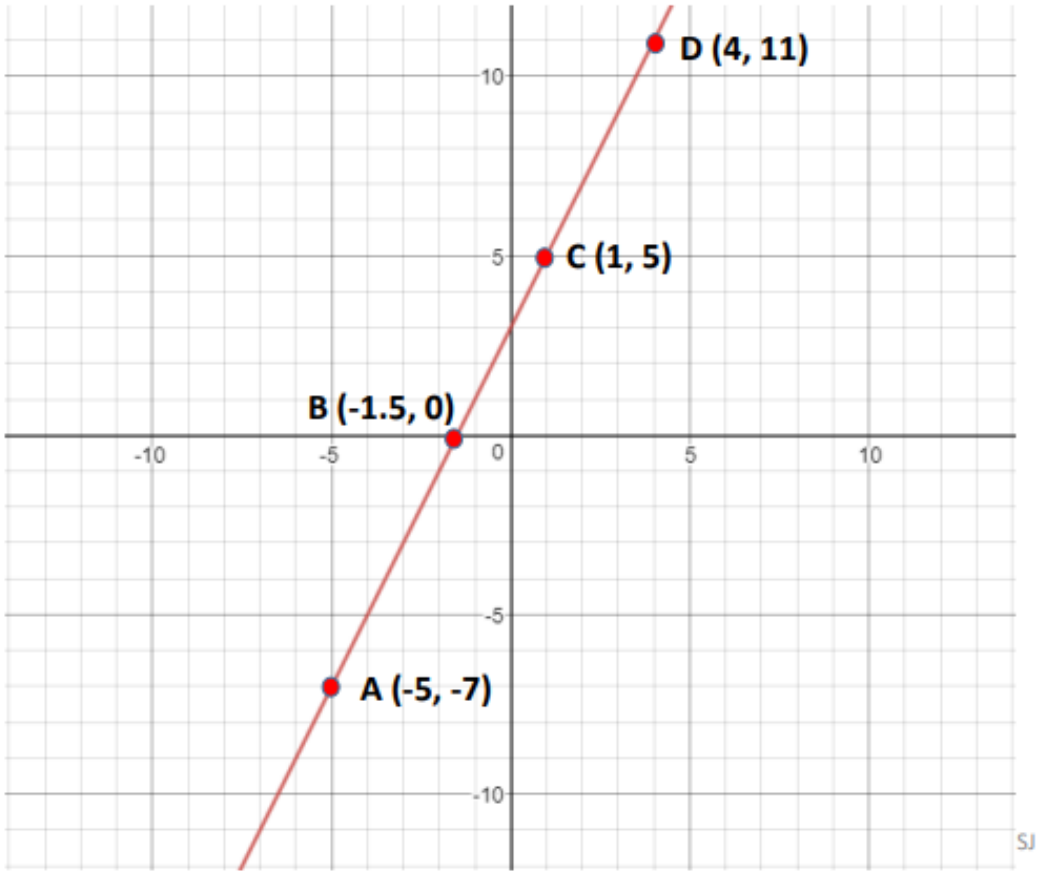
How you find gradient of a straight line ??

$$\text{Slope} = \frac{\text{Change in Y}}{\text{Change in X}}$$



Differentiation: Gradient of a straight line

Consider the following graph of the linear function $y = 2x + 3$



What is the gradient of **AB, BC and CD**?

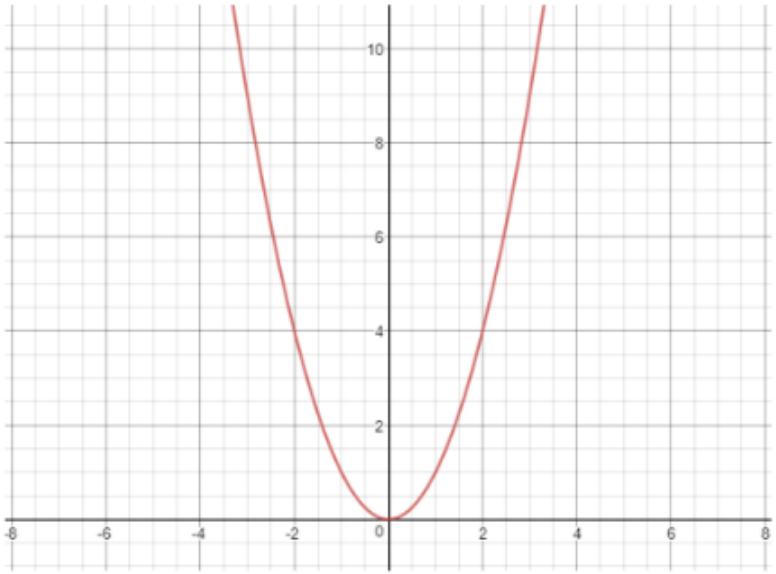
m_{AB} = 2

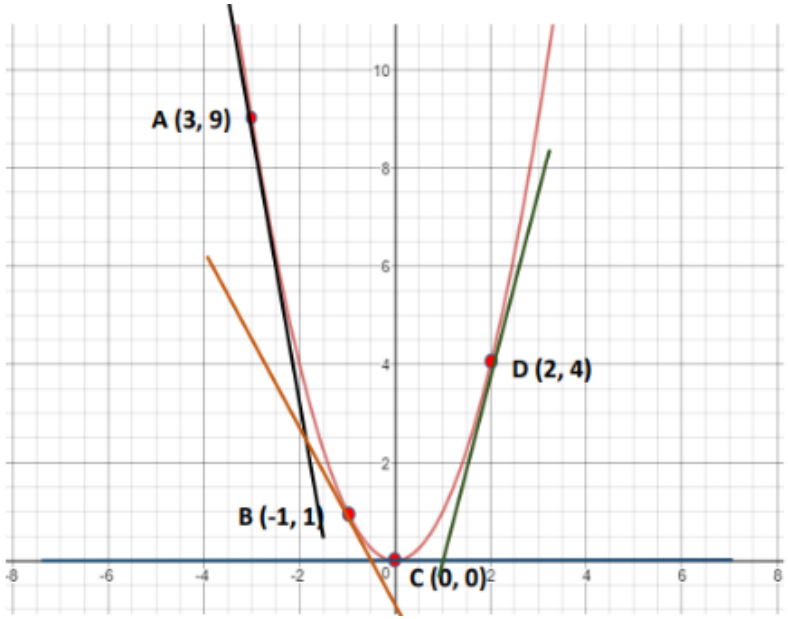
m_{BC} = 2

m_{CD} = 2

Differentiation: Gradient of a curve

Consider the following graph of the non-linear function $y = X^2$





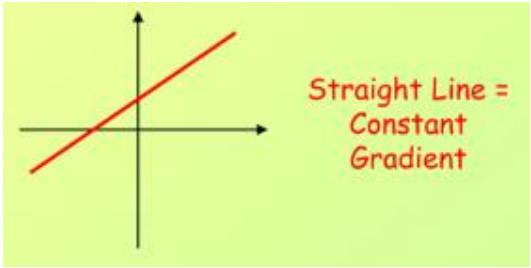
What can you say about the gradient of this curve at different sections along the curve?

Simulating Gradient of a curve

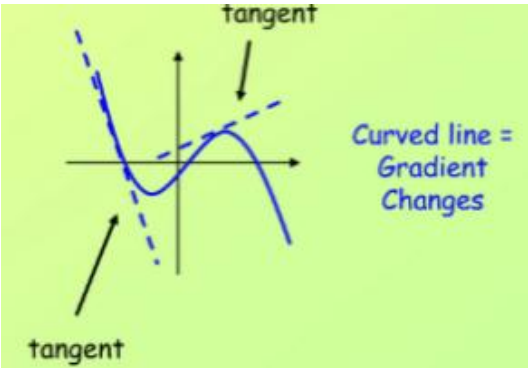
https://www.geogebra.org/m/hbjzvjpv

Differentiation: : Definition of derivative

- On a straight-line graph, **the gradient is constant, the same everywhere along the line**.

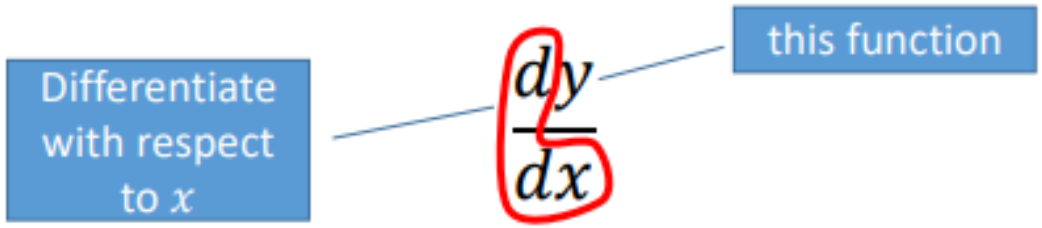


- On any curved graph, **the gradient is always changing from point to point.** Its value depends on where you are along the x-axis.



Differentiation is the process by which you can find the **"gradient function"** of a function **"at a point"**. When you differentiate a function, you get its gradient function which is formally called the **derivative function** or just **derivative.**

Differentiation: Notation



function

$f(x)$

Differentiate

derivative function

$f'(x)$

y

Differentiate

$\frac{dy}{dx}$

Differentiation : Basic rules

	Function $f(x)$	Derivative function $f'(x)$
Constant rule	$f(x) = k$ k *is any real number*	$f'(x) = 0$
Power rule	$f(x) = x^n$ n *is any real power*	$f'^{(x)} = nx^{n-1}$
e rule	$f(x) = e^x$	$f'(x) = e^x$
***ln* rule**	$f(x) = \ln x$	$f'(x) = \frac{1}{x}$
Sin rule	$f(x) = \sin x$	$f'(x) = \cos x$
Cos rule	$f(x) = \cos x$	$f'(x) = -\sin x$
Tan rule	$f(x) = \tan x$	$f'(x) = \sec^2 x$

Differentiation : Power Rule Examples

Find the derived function of each of the following

$f_{(x)} = x^3$ >> $f'_{(x)} = 3x^2$

$f_{(x)} = x^{-2}$ >> $f'_{(x)} = -2x^{-3}$

$f_{(x)} = x^{-\frac{1}{4}}$ >> $f'_{(x)} = -\frac{1}{4}x^{-\frac{5}{4}}$

Differentiation : Power Rule Examples

Find the derived function of each of the following

$f_{(x)} = \frac{1}{x^2}$ >>

Must be written in ax^n →

$f_{(x)} = \frac{1}{x^2}$

$f_{(x)} = x^{-2}$

$f'_{(x)} = -2x^{-3}$

$f_{(x)} = \sqrt{x}$ >>

Must be written in ax^n →

$f_{(x)} = \sqrt{x}$

$f_{(x)} = (x)^{\frac{1}{2}}$

$f'_{(x)} = \frac{1}{2}x^{-\frac{1}{2}}$

Differentiation : Basic Rules

	Function y	Derivative function $\frac{dy}{dx}$
Constant multiple rule	$y = kf(x)$	$\frac{dy}{dx} = kf'(x)$
Sum rule	$y = f(x) + g(x)$	$\frac{dy}{dx} = f'(x) + g'(x)$
Difference rule	$y = f(x) - g(x)$	$\frac{dy}{dx} = f'^{(x)} - g'(x)$

Differentiation : Derivatives of polynomial functions

Find the derived function of each of the following

$y = 2x^3$ >> $y = 2 \times f(x^3)$

Constant Multiple rule

$$\frac{dy}{dx} = 2 \times f'(x^3)$$

$$\frac{dy}{dx} = 2 \times 3x^2$$

Power rule

$$\frac{dy}{dx} = 6x^2$$

Differentiation : Derivatives of polynomial functions

Find the derived function of each of the following

$$y = 3 + 4x^2 \quad >> \quad y = f(3) + 4 \times g(x^2)$$

Sum rule

$$\frac{dy}{dx} = f'(3) + 4 \times g'(x^2)$$

Constant Multiple rule

$$\frac{dy}{dx} = 0 + 4 \times 2x^1$$

Power rule

Constant rule

$$\frac{dy}{dx} = 8x$$

Differentiation : Derivatives of polynomial functions

Find the derived function of each of the following

$$y = 2 - 5x^3 \quad >> \quad y = f(2) - 5 \times g(x^3)$$

Difference rule

$$\frac{dy}{dx} = f'(2) - 5 \times g'(x^3)$$

Constant Multiple rule

$$\frac{dy}{dx} = 0 - 5 \times 3x^2$$

Power rule

Constant rule

$$\frac{dy}{dx} = -15x^2$$

Differentiation : Evaluating the gradient at any point

Once the $\frac{dy}{dx}$ is found, it can be used to find the gradient at any point $x = x_0$.

When the gradient is evaluated at a point x_0, it's written as,

$$f'(x_0) \quad or \quad \frac{dy}{dx}_{x=x_0}$$

Differentiation : Evaluating slope at a point Example

Calculate the gradient of the curve $y = 4x^2 - 8x + 3$ at coordinate (-2,4)

$$y = 4x^2 - 8x + 3 \quad >> \quad y = 4 \times f(x^2) - 8 \times g(x) + h(3)$$

$$\frac{dy}{dx} = 4 \times f'(x^2) - 8 \times g'(x) + h'(3)$$

$$\frac{dy}{dx} = 8x - 8$$ ← **Derivative**

$$\frac{dy}{dx}_{x=-2} = 8 \times (-2) - 8$$ ← **Substitute in x = -2**

$$\frac{dy}{dx}_{x=-2} = -24$$ ← **Gradient/Slope**

Thank You!